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Notes on giant gravitons on PP-waves

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Abstract: We investigate the giant gravitons in the maximally supersymmetric IIB pp-wave from several viewpoints: (i) the dynamics of D3-branes, (ii) the world-sheet description and (iii) the correlation functions in the dual $N = 4$ Yang-Mills theory. In particular, we derive the BPS equation of a D3-brane with magnetic flux, which is equivalent to multiple D-strings, and discuss the behavior of solutions in the presence of RR-flux. We find solutions which represent the excitations of the giant gravitons in that system.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Penrose limit and pp-wave background.

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Contents

1. Introduction

Recently, it was shown that the string theory on the maximally supersymmetric type-IIB pp-wave [1] can be obtained by taking the Penrose limit of the familiar $AdS_5 \times S^5$ [2]. Furthermore, the string theory on the pp-wave turns out to be exactly solvable [3, 4]. Thus it is possible to check the AdS/CFT duality including the stringy modes and indeed this correspondence between the gauge theory operators and perturbative closed string states has been successfully compared [5]. However, there are also non-perturbative objects of D-branes which survive this limit and thus should also have the gauge duals. One of the important such examples is the giant graviton, which is described by a D3-brane wrapped on S^3 [6]–[8]. The dual operator in the $N = 4$ Super Yang-Mills theory was proposed in [9] and we can compute the correlation function exactly [10]. As we will see later, the correlation function is finite in the large N limit only if we assume the scaling of Penrose limit. Therefore we believe that the study of giant gravitons is important to know the duality between the pp-wave and the gauge theory well.

D-branes in pp-wave backgrounds can be classified into two groups (for recent studies see [11]–[31]). One of them can be described by the light-cone world-sheet theory [11, 12, 17, 23] and the other cannot [17, 24, 25]. The Penrose limit of giant gravitons (spherical D3-brane) is also divided into the two types above. The open string spectrum of the

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former D-branes can be exactly examined and its comparison to the gauge theory has been studied [13, 15, 17, 18]. In the light-cone gauge naively one may think that the D-brane cannot move away from the origin because of the mass term. However, as we will show later, some of the D-branes can move without breaking supersymmetry if we assume the appropriate boosting or gauge flux (for related previous arguments see also [17, 27]). In the case of a D3-brane this just corresponds to the shift of the radius of giant gravitons.

On the other hand, even though the latter has no such stringy description, the D-brane in this group has the interesting aspect that they can couple to the background RR-field directly. For example, the D3-brane can be expanded into its spherical form due to the RR-flux, which is identified with the giant graviton. Another interesting example is the system of multiple D-strings. This can couple to the RR-flux via the non-abelian term (Myers term [32]). As we will see in this paper, they can expand without losing any energy. We will also investigate the system from the viewpoint of a D3-brane and show that it is a 1/4 BPS state. These moduli of multiple D-strings are argued to be excitations of giant gravitons.

This paper is organized as follows. In section 2 we discuss the world-volume of giant graviton in the Penrose limit and also examine the correlators of its dual Yang-Mills operator. In section 3 we investigate the dynamics of multiple D1-branes and show that there are BPS excitations of giant gravitons. In section 4 we analyze the world-sheet description of giant gravitons and derive the supersymmetric condition. In the appendix we present the detailed computations of the BPS condition of a D3-brane with magnetic gauge flux in the pp-wave background.

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2. Giant gravitons in PP-wave background

The maximally supersymmetric pp-wave background[1] is given by

$$\begin{aligned} ds^2 &= -2\,\mathrm{d}x^+\,\mathrm{d}x^- - \mu^2\left(\sum_{I=1}^{8} x^I x^I\right)\left(\mathrm{d}x^+\right)^2 + \sum_{I=1}^{8}\left(\mathrm{d}x^I\,\mathrm{d}x^I\right), \\ F_{+1234} &= F_{+5678} = 4\mu\,. \end{aligned} \tag{2.1}$$

We can obtain this background from $AdS_5 \times S^5$

$$ds^2 = R_A^2\left(-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho\; d\Omega_3^2 + d\psi^2\cos^2\theta + d\theta^2 + \sin^2\theta d\Omega_3^{\prime 2}\right), \tag{2.2}$$

where the radius is given by $R_A = (4\pi g_s N\alpha'^2)^{1/4}$, by taking the Penrose limit $R_A \to \infty$ [2] scaling as

$$x^+ = \frac{(t+\psi)}{2\mu}\,, \qquad x^- = R_A^2\mu(t-\psi)\,, \qquad \rho \to \frac{\rho}{R_A}\,, \qquad \theta \to \frac{\theta}{R_A}\,. \tag{2.3}$$

[1] In this paper we will use the notation of [4] (but slightly different). In our convention the SUGRA equation of motion is given by $e^{-2\phi}R_{\mu\nu} = (1/96)F_{\alpha\beta\gamma\delta\mu}F_\nu^{\alpha\beta\gamma\delta}$. This is the same as in Polchinski's text book [33] and has a merit that WZ-term of D-branes is normalized conventionally such that $S_{WZ} = \mu_p\int_{D_p} C_{(p)}$ $\;(\mu_p = T_p e^{\phi})$.

We also define the polar coordinate of S^5 ($Y_1^2+Y_2^2+Y_3^2+Y_4^2+Y_5^2+Y_6^2=R_A^2$) as follows

$$\begin{aligned} Y_1 &= R_A\cos\theta\sin\psi\,, & Y_2 &= R_A\cos\theta\cos\psi\,, & Y_3 &= R_A\sin\theta\cos\alpha\cos\beta\,, \\ Y_4 &= R_A\sin\theta\cos\alpha\sin\beta\,, & Y_5 &= R_A\sin\theta\sin\alpha\cos\gamma\,, & Y_6 &= R_A\sin\theta\sin\alpha\sin\gamma\,. \end{aligned} \tag{2.4}$$

2.1 Giant gravitons in the PP-wave background

Here we would like to consider giant gravitons (D3-branes) [6] in the Penrose limit. A giant graviton is a D3-brane in the $AdS_5\times S^5$ background which is rotated in the angular direction and is expanded due to the force of RR-field. Its spatial world-volume is given by S^3 in S^5 [6] or AdS_5 [7, 8]. Since in the pp-wave background we have the $\mathbb{Z}_2$ symmetry which interchanges $x^5\sim x^8$ with $x^1\sim x^4$ (or equally in the AdS language S^5 with AdS_5), the physical properties of these two should be the same. Thus we only discuss the first type of giant graviton here. Later we will mention that it is not obvious, however, to see this symmetry in the gauge dual picture.

Now, the resulting D3-branes in the Penrose limit can be largely divided into two groups. One of them is a spherical D3-brane whose would-volume is taken in the direction x^+ and $S^3(\in S^5)$. After we take the limit, this giant graviton is given by

$$t=\psi\,,\qquad a=R_A\theta(=\text{fixed})\,, \tag{2.5}$$

where a is equal to the radius of S^3.

Another configuration is the D3-branes which have Neumann boundary conditions in the $x^\pm$, x^i and x^j direction ($i,j\in\{1,2,3,4\}$ or $i,j\in\{5,6,7,8\}$). In order to see this in detail, let us consider the giant gravitons whose world-volume is given by

$$Y_1^2+Y_2^2+Y_3^2+Y_4^2={R'_A}^2\,,\qquad \gamma=t\,. \tag{2.6}$$

Next we take the Penrose limit $R_A\to\infty$ with keeping r finite, where r is defined by

$$\left(\frac{R'_A}{R_A}\right)^2=1-\left(\frac{r}{R_A}\right)^2\,. \tag{2.7}$$

Then we obtain[2]

$$\begin{aligned} x^7 &= \sqrt{y^2-r^2}\cos\beta\,, & x^8 &= \sqrt{y^2-r^2}\sin\beta\,, \\ x^5 &= r\cos(\mu x^+)\,, & x^6 &= r\sin(\mu x^+)\,, \end{aligned} \tag{2.8}$$

where we define the parameter $y=\theta R_A$. Since θ and β are free parameters, the resulting D-brane has the Neumann boundary condition in the direction x^7, x^8 and the Dirichlet in x^5, x^6. Thus this represents a D3-brane placed away from the origin by r. Since the coordinates depend on x^+, this brane is rotating in the x^5-x^6 plane. If it does not rotate, then it cannot preserve sixteen supersymmetries, as we will see later.

Below we first discuss the former type of giant gravitons. The latter type allows the world-sheet analysis in the light-cone gauge and will be studied in section 4.

[2] Obviously we can shift the light-cone time as $x^+\to x^+ +\text{const.}$ without changing physics.

2.2 Comments on correlators of giant gravitons in PP-wave limit

Here we would like to briefly comment on the gauge dual of the first type[3] of giant gravitons in the Penrose limit employing the proposal of the dual Yang-Mills operators discussed in [9].

In the paper [9] the gauge theoretic dual of the giant graviton with angular momentum J which expands in the S^5 direction is conjectured to be the BPS subdeterminant operator with U(1) R-charge J

$$O_J^{S^5} = \frac{1}{J!(N-J)!}\epsilon_{a_1 a_2 \cdots a_N}\epsilon^{b_1 b_2 \cdots b_J a_{J+1}\cdots a_N} Z^{b_1}_{a_1} \cdots Z^{b_J}_{a_J} \,. \tag{2.9}$$

The radius a of the corresponding giant gravitons is given by

$$a = \sqrt{\frac{J}{N}} R_A \,. \tag{2.10}$$

In particular the maximal one is described by the determinant operator. Then by using the results and technique found in [10], we can compute various (normalized) correlators of giant gravitons exactly.[4] Now we are interested in the Penrose limit $J \sim O(\sqrt{N})$ and then the normalized three point function[5] in the $N=4$ U(N) gauge theory is given by

$$\langle O_J^{S^5} \bar{O}_{J_1}^{S^5} \bar{O}_{J_2}^{S^5} \rangle = \sqrt{\frac{(N-J_1)!(N-J_2)!}{(N-J)!N!}} \simeq e^{-J_1 J_2/2N} \,. \tag{2.11}$$

This result[6] shows that the single particle state of (2.9) is well defined (i.e. not mixed with multiparticle states) if we assume[7] $J^2/N \gg 1$. Thus the condition that the description of BPS operator (2.9) should describe the geometrical object of giant graviton requires at least $J^2/N \gg 1$. It may also be useful to note that the similar behaviour of the correlator was found in the pp-wave wave limit of $AdS_3 \times S^3$ [36] (see also [37]).

Next let us compute the overlap between a single trace operator and (2.9) in the limit $J \sim O(\sqrt{N})$

$$\langle \mathrm{Tr} Z^J \ \bar{O}_J^{S^5} \rangle \simeq \frac{e^{-(J^2/4N)}}{\sqrt{J}} \sqrt{\frac{J^2/2N}{\sinh(J^2/2N)}} \,. \tag{2.12}$$

To derive the above result[8] we have used the fact that the operator $\mathrm{Tr}Z^J$ can be represented as a sum of Schur functions $\chi_h(Z)$, which is exactly the same as the operator (2.9), such

[3] The gauge dual of the latter type, which allows the exact world-sheet analysis, was discussed in [18] and the agreement of anomalous dimension of their fluctuations is verified (see also [34] for BPS fluctuations).

[4] There are several results on the non-renormalization of extremal correlators e.g. [35] even for multitrace operators. Thus the results do not receive no g_{YM} corrections. We would like to thank S.Ramgoolam very much for this argument.

[5] The two point function, which is used to normalize other correlation functions, is given by $\langle \bar{O}_J^{S^5} O_J^{S^5} \rangle = (N!/(N-J)!) \sim N^J e^{-J^2/2N}$.

[6] The exponential behaviour may suggest a sort of a tunneling effect. We would like to thank S.Minwalla very much for suggesting this point and also other several useful comments.

[7] More precisely, here we mean that J^2/N is always finite taking the limit $N \to \infty$ and that the value J^2/N is enough larger than one.

[8] The factor of $\sinh(J^2/2N)$ comes from the the two point function [38, 39] $\langle \mathrm{Tr}Z^J \mathrm{Tr}\bar{Z}^J \rangle = JN^J \times (\sinh(J^2/2N)/J^2/2N)$, which is used to normalize.

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that $\mathrm{Tr}Z^J = \sum_h \pm\chi_h(Z)$ (here h denotes all of the hooks of Young diagram and for more detail see the appendix of the first paper of [38]).

Let us try to interpret this result qualitatively from the viewpoint of the giant graviton in the string theory side. The single trace operator is regarded as a light-cone vacuum $|p^+\rangle$ following the arguments in [5] and the wave function in the supergravity is given by $\varphi \sim e^{-\mu p^+ x^2}$. We also assume that the giant graviton has the width of δa by quantum effects such that the wave function of D3-branes is estimated by $\varphi' \sim e^{-((x-a)^2/(\delta a)^2)}$. Then it is easy to see that the overlap (2.12) will be proportional to $\exp(-((J^2/N)/((J(\delta a)^2/R_A^2)+1)))$. Thus we can find the scaling

$$\delta a \sim \frac{R_A}{\sqrt{J}} = \left(4\pi\alpha'^2 \frac{g_s N}{J^2}\right)^{1/4} \quad (\ll a)\,, \tag{2.13}$$

where we roughly examined neglecting any finite constant factor only in this part. The width becomes zero if we neglect quantum effect $g_s = 0$ as is expected. Also note that the expression depends only on the effective coupling $(Ng_s)/J^2$. By using this result we can reproduce the exponential scaling behaviour of the three point function (2.11). Then we can argue that the description of the giant graviton by using the operator (2.9) is good[9] if

$$(\delta a)^2 \ll \alpha' \ll a^2\,. \tag{2.14}$$

There are also arguments on a giant graviton which expands in the AdS_5 direction [8, 10]. The authors of [10] conjectured that it corresponds to the operator

$$O_J^{AdS_5} = \frac{1}{J!} \sum_{\sigma\in\mathcal{S}_J} Z^{a_1}_{a_{\sigma(1)}} Z^{a_2}_{a_{\sigma(2)}} \cdots Z^{a_J}_{a_{\sigma(J)}}\,, \tag{2.15}$$

where $\mathcal{S}_J$ denotes the permutation group of length J.

We can take the pp-wave limit for the normalized three point function obtained in [10] as follows

$$\langle O_J^{AdS_5} \bar{O}_{J_1}^{AdS_5} \bar{O}_{J_2}^{AdS_5}\rangle = \sqrt{\frac{(N+J-1)!(N-1)!}{(N+J_1-1)!(N+J_2-1)!}} \simeq e^{J_1 J_2/2N}\,. \tag{2.16}$$

The overlap between a single trace operator and $O_J^{AdS_5}$ can also be computed in the same way as in (2.12)

$$\langle \mathrm{Tr}Z^J \bar{O}_J^{AdS_5}\rangle \simeq \frac{e^{(J^2/4N)}}{\sqrt{J}} \sqrt{\frac{J^2/2N}{\sinh(J^2/2N)}}\,. \tag{2.17}$$

As can be seen from these, both become too large when $J^2/N \gg 1$. Thus the expression (2.15) seems not be a good quantum state in our scaling limit. There is one more strong evidence for this. In the pp-wave background there is the $\mathbb{Z}_2$ symmetry which exchanges $x^1 \sim x^4$ with $x^5 \sim x^8$ and we should observe this symmetry in our correlation functions. However, our results do not show such a behaviour.

[9]The latter bound represents the critical point where gravitons become giant as discussed in [5].

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In the above we have examined several correlators of the gauge dual operators of giant gravitons and found the exponential behaviour in the pp-wave limit with respect to the ones expanded in the S^5 direction. Even though we gave a quantitative interpretation of this result, the complete explanation requires the computation of the interaction between the curved D-branes, which is not tractable at present and will be left as a future problem. The analysis of dual operators of giant gravitons in AdS_5 seems to be much more difficult.

3. Giant gravitons in multiple D1-branes on PP-wave

Next we would like to investigate giant gravitons in the pp-wave from the viewpoint of lower dimensional D-branes (D1-branes). Let us consider the D1-brane extended in the direction $x^+(\equiv \tau)$ and $x^4(\equiv y)$. The metric of its world-volume is given by

$$ds^2 = -\mu^2 y^2 (dx^+)^2 + dy^2\,. \tag{3.1}$$

This kind of D1-branes were first discussed in [17] and shown to be 1/4 BPS. It is easy to check that it satisfies the equation of motion for DBI action in the pp-wave background. Below we are interested in the static solutions of the non-abelian system of many D1-branes[10] (the number of D1-branes is defined to be $M(\gg 1)$).

3.1 Non-abelian dynamics of multiple D1-branes on PP-wave

The transverse scalars are defined by $\Phi^i = (2\pi\alpha')^{-1}x^i$ $(i = 1,2,3,5,6,7,8)$. Below we only consider $\vec{\Phi} = (\Phi^1, \Phi^2, \Phi^3)$ because the other scalars will not play any interesting role in the presence of the RR-field $C_{+124} = -4\mu x^3$ via the RR couplings [32] $\mu_p \int \exp(i2\pi\alpha'\iota_\Phi\iota_\Phi) \wedge C \sim \int \mathrm{Tr}[C_{+ijk}[\Phi^i,\Phi^j]\partial_y\Phi^k]$. Then the non-abelian action which describes our static configuration is given by

$$\begin{aligned} S = &-T_{D1}\int d\tau dy \mathrm{STr}\left[\sqrt{\mu^2(y^2+(2\pi\alpha')^2|\vec{\Phi}|^2)(1+(2\pi\alpha')^2\partial_y\Phi^i Q^{-1}_{ij}\partial_y\Phi^j)(\det Q_{ij})}\right] \pm \\ &\pm T_{D1}\int d\tau dy \mathrm{Tr}[(16i\pi^2{\alpha'}^2\mu)[\Phi^1,\Phi^2]\Phi^3]\,, \end{aligned} \tag{3.2}$$

where we defined $Q_{ij} = \delta_{ij} + i(2\pi\alpha')[\Phi^i,\Phi^j]$ and the D-brane tension $T_{Dp} = 1/((2\pi)^p g_s \times (\alpha)^{(p+1)/2})$. The symbol STr denotes the symmetric trace [43]. The last term represents the effect of RR-flux [32] and the $\pm$ sign in front of it depends whether we consider branes or antibranes. We can choose the minus sign without losing the generality. From this action it is obvious that the constant shift of the transverse scalars is not the moduli of the system and it lifts the energy. We are interested in the way how to get the supersymmetric solutions with non-zero expectation values of the transverse scalars as shown below.

Since the full non-abelian action is too complicated, first we would like to assume

$$\alpha'|\vec{\Phi}| \ll y\,, \qquad \sqrt{\alpha'}|\vec{\Phi}| \ll \sqrt{M}\,, \qquad \alpha'|\partial_y\vec{\Phi}| \ll 1\,, \tag{3.3}$$

[10] For the relevant discussion of multiple D-string in flat space see [40, 41]. See also [42] for discussions on related non-abelian dynamics of D-strings in more general or other examples.

which restricts the value of $\vec{\Phi}$ at $y = 0$ to zero. Later we will return to more general solutions.

Then we can approximate the action (3.2) as follows

$$S \cong (\pi\alpha')^2 T_{D1} e^{-\phi} \int d\tau dy \mathrm{Tr}\left[-2\mu|y||\partial_y\vec{\Phi}|^2 - \frac{2\mu}{|y|}|\vec{\Phi}|^2 - 16i\mu[\Phi^1,\Phi^2]\Phi^3 + \sum_{i,j}\mu|y|[\Phi^i,\Phi^j]^2\right], \tag{3.4}$$

where we omit the constant term $-\mu T_{D1}\int dy|y|$. The validity of this approximation (3.3) will be confirmed later. We obtain the Hamiltonian as follows

$$H = \mu\pi^2\alpha'^2 T_{D1}\int dy \mathrm{Tr}\left[2|y||\partial_y\vec{\Phi}|^2 + \frac{2}{|y|}|\vec{\Phi}|^2 + \frac{8}{3}i\epsilon_{ijk}[\Phi^i,\Phi^j]\Phi^k - |y|[\Phi^i,\Phi^j]^2\right]. \tag{3.5}$$

We can rewrite the expression (3.5) as a sum of squares and boundary terms

$$H = \mu\pi^2\alpha'^2 T_{D1}\int dy \;\; \mathrm{Tr}\left[\frac{2}{|y|}\left(y\frac{d\Phi^i}{dy} - \Phi^i - \frac{i|y|}{2}\epsilon_{ijk}[\Phi^j,\Phi^k]\right)^2 + \right.$$
$$\left. + \frac{\partial}{\partial y}\left(2\frac{y}{|y|}\Phi^i\Phi^i + \frac{2iy}{3}\epsilon_{ijk}[\Phi^i,\Phi^j]\Phi^k\right)\right], \tag{3.6}$$

where we have used the previous constraint $\Phi^i(0) = 0$. In this way we obtain the following BPS equation (or[11] 'Nahm like equation in the pp-wave background')

$$\frac{d\Phi^i}{dy} = \frac{i}{2}\frac{y}{|y|}\epsilon_{ijk}[\Phi^j,\Phi^k] + \frac{1}{y}\Phi^i. \tag{3.7}$$

To find its solution we assume the following ansatz

$$[\Phi^i,\Phi^j] = i\epsilon_{ijk}R(y)\Phi^k, \tag{3.8}$$

that is the fuzzy sphere structure [45]. Since we are interested in a bound state of M D1-branes, we have only to consider the M dimensional irreducible representation of the SU(2) algebra. In the most of the discussion below we assume M is large. Then its trace is normalized such that

$$\sum_{i=1}^{3}\mathrm{Tr}(\Phi^i\Phi^i) = \frac{(M-1)M(M+1)}{4}R(y)^2, \tag{3.9}$$

and thus the radius r of the sphere is given by

$$r = 2\pi\alpha'\sqrt{\frac{1}{M}\mathrm{Tr}[\Phi^i\Phi^i]} = \sqrt{M^2-1}\pi\alpha' R \sim M\pi\alpha' R. \tag{3.10}$$

Now, the nontrivial solutions of (3.7) are obtained as follows

$$R(y) = \frac{2C|y|}{1+Cy^2}, \tag{3.11}$$

[11] See also [44] for a different appearance of Nahm like equation from the D1-branes [40, 41] in AdS background, which is not related to our D1-branes directly.

where C is an arbitrary constant. If C is negative, then the value of R diverges at some points and this solution may seem to represent a kind of a spike solution. However, in this case the approximation (3.3) is violated and indeed later we will see that there is no such BPS solution from the view point of a D3-brane with magnetic flux.

Therefore we regard C as a positive constant. Then we find the smooth solution in $-\infty < y < \infty$. Its energy is given by zero (see (3.6)) and thus it is expected to be stable and BPS. In other words the deformation of the original D1-branes by the parameter C should be the moduli of supersymmetric vacua. Though it is not easy to check the supersymmetry in this non-abelian system, we will show that this is the case in the analysis of the D3-brane. Our approximation of (3.3) is valid if $MC\alpha' \ll 1$ for the BPS solution (3.11) (remember we are interested in the limit of $M \gg 1$.) Then the maximal radius of the fuzzy sphere can also be estimated $r \sim \pi\alpha' M\sqrt{C}$ and can take larger values than the string scale. Thus we can believe this solution is physically relevant. We would also like to note that all the computation above does not use the condition $M \gg 1$ and even if we do not assume it, the solution (3.11) may be meaningful.

Next we discuss the interpretation of this classical solution. Since the M D1-branes can expand into a D3-brane due to Myers effect [32], we can speculate that the configuration (3.11) is a BPS bound state of M D1-branes and two D3-branes each located at $y = \pm 1/\sqrt{C}$. These two D3-branes can be regarded as a giant graviton with the small radius $r \ll \sqrt{M\alpha'}$ at the origin divided into two parts. As we will see in the next subsection 3.2 we can show that the 1/4 BPS solutions of a D3-brane with magnetic flux include this solution. Our result that the energy of this system is the same as that of M D1-branes is also consistent with the fact that giant gravitons have the vanishing value of $p^- = \mu(\Delta - J)$.

3.2 BPS configurations of D3-brane in PP-wave background

Now one may ask what will happen if we consider larger giant graviton $r \gg \sqrt{M\alpha'}$ in the pp-wave background. In order to examine them we must return to the full non-abelian action (3.2). Since we assume $M \gg 1$, the symmetric trace STr can be replaced with the ordinary trace Tr. Applying the ansatz (3.8), we can rewrite the Hamiltonian of the action (3.2) as follows ($\alpha^2 \equiv (2\pi\alpha')^2(M^2-1)$)

$$H = M\mu T_{D1}\int dy \left[\sqrt{\left(y^2 + \frac{\alpha^2}{4}R^2\right)\left(1 + \frac{\alpha^2}{4}R^4\right)\left(1 + \frac{\alpha^2}{4}\left(\partial_y R\right)^2\right)} - \frac{\alpha^2}{3}R^3\right]. \quad (3.12)$$

This action just coincides with the effective action (DBI+Chern-Simons term) of a D3-brane with the magnetic flux on S^2 (fuzzy sphere)

$$F = (\pi\alpha' M)\sin\theta d\theta \wedge d\varphi\,, \quad (3.13)$$

in the pp-wave background. Then we can derive the supersymmetric condition (BPS equation) of this system since we have only to consider a single D-brane. As we show the detailed analysis in the appendix, we can verify that the system keeps eight dynamical supersymmetries (1/4 BPS), which is the same number in D1-branes, if the following BPS equation is satisfied (see (A.13) and use the relation $\rho = \alpha R/2$)

$$\frac{dR}{dy} = \frac{\pm R^2 y + R}{y \mp \frac{\alpha^2}{4} R^3}\,, \tag{3.14}$$

where we should choose $+$ $(-)$ sign in the right-hand side if $y + \frac{\alpha^2}{4} R\,\partial_y R < 0$ (> 0). We can rewrite this as $y - \frac{\alpha^2}{4}R^3 < 0$ $(y + \frac{\alpha^2}{4}R^3 > 0)$ by using the BPS equation. The existence of dynamical supersymmetries Q^- is very important because we now regard the energy as the light-cone energy $p^-(\sim \{Q^-, Q^-\})$. Note also that if we assume the approximation $|y| \gg \frac{\alpha^2}{4}R^3$ (or equally $MC\alpha' \ll 1$ as before), we get the previous BPS equation

$$\frac{dR}{dy} = \pm R^2 + \frac{R}{y}\,. \tag{3.15}$$

This just matches with our previous result of non-abelian D1-branes. If we consider the D3-brane with no magnetic flux $M = 0$, then we can also see that the solution is given by a three dimensional sphere $\rho^2 + y^2$ =constant and this is consistent with the result in [5, 17].

This BPS equation can also be obtained from the viewpoint of the action of a D3-brane (or equally multi D-strings) (3.12). For this aim the crucial identity is

$$\begin{aligned}\left(y^2+\frac{\alpha^2}{4}R^2\right)\left(1+\frac{\alpha^2}{4}R^4\right)\left(1+\frac{\alpha^2}{4}(\partial_y R)^2\right) &= \left(\frac{\alpha^2}{4}(yR^2R' - R^3) \pm \left(y + \frac{\alpha^2}{4}RR'\right)\right)^2 + \\ &\quad + \frac{\alpha^2}{4}\left(\left(yR^2 + \frac{\alpha^2}{4}R^3R'\right) \mp (yR' - R)\right)^2 .\end{aligned}$$

Note that the second term is the square of BPS eq. (3.14). Then the hamiltonian (3.12) is estimated as follows (we restore μ)

$$H \geq \frac{\alpha^2 M}{4} T_{D1}\mu \int dy \frac{d}{\partial y}\left(-\frac{1}{3}R^3 y \mp \frac{1}{2}R^2\right) \mp MT_{D1}\mu \int dy\ y. \tag{3.16}$$

The equality holds only when BPS eq. (3.14) is satisfied. The second term represents the energy of M D1-branes with $\pm$ signs. If we assume that $R(0) = 0$, $\lim_{y\to\infty} yR(y)^3 = 0$ and $\lim_{y\to\infty} R(y)^2 = 0$ (the second and third condition mean that we consider a solution which is asymptotically D1-branes), then we obtain[12]

$$\begin{aligned}&H - MT_{D1}\mu \int dy|y| \geq \\ &\geq \frac{\alpha^2 M}{4} T_{D1}\mu \left[\int_0^\infty dy \frac{d}{\partial y}\left(-\frac{1}{3}R^3 y + \frac{1}{2}R^2\right) + \int_{-\infty}^0 dy \frac{d}{\partial y}\left(-\frac{1}{3}R^3 y - \frac{1}{2}R^2\right)\right] \\ &= 0\,.\end{aligned} \tag{3.17}$$

Thus in this case all of the BPS solutions have the same energy as M D1-branes. In other words this is the moduli of the system.

[12]This is because the condition $y - (\alpha^2/4)R^3 < 0$ $(y + (\alpha^2/4)R^3 > 0)$ is equivalent to $y < 0$ $(y > 0)$ if $R(0) = 0$.

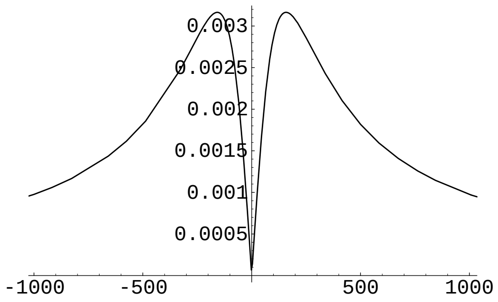

Figure 1: The behaviour of a solution $R(y)$ in the case $R_{\max}(= 0.032) < R_0$. Here we scaled y and $R(y)$ such that $\alpha = 1$.

Now we would like to classify the solutions to the BPS equation (3.14). After some numerical study we find that there are two types of solutions. The first type is well approximated by the previous solutions (3.11). This exists only when $R_{\max} < R_0$, where R_0 is a constant of order $\sim \alpha^{-1/2}$. See figure 1 below. All of these solutions have the zero energy as we can see from (3.17). Note that $R(0) = 0$ holds for them and they are symmetric under $y \to -y$.

Another type of solutions is possible for $R_{\max} > R_0$ and this is not exactly symmetric under $y \to -y$. This includes a large lump at the origin, which is regarded as a giant graviton. See figure 2 below. The remarkable result is that this solution has less energy than the original D1-branes. Thus this implies that the system of multiple D1-branes may be unstable, while a single D1-brane is stable. The 'instability' observed here is closely related to the fact that even though the eight Killing spinors always are preserved in the D3-brane system, they are not continuous at the point $y + (\alpha^2/4)R^3 = 0$ as the $\pm$ sign in (A.12) flips. In this sense the 1/4 BPS system of the D3-brane in the pp-wave background is somewhat different from the usual BPS state. This nature of Killing spinors also equally applies to a single D1-brane in the pp-wave background in the same way even though this is not mentioned before. In a sense we may say that the D3-brane (or D1-brane) turns into anti D3-brane (anti D1-brane) at the point $y + (\alpha^2/4)R^3 = 0$. However, we have some doubts whether this solution is physical or not because for this solution we have the unnatural discontinuity[13] of the differential of R at the point $y + (\alpha^2/4)R^3 = 0$. We would like to leave the correct interpretation of this solution as a future problem. If this instability is true, then our result tells that a system of multiple D1-branes will decay to a giant graviton, which has the zero energy.

4. World-sheet description of giant gravitons

In this section we investigate those BPS D-branes on the pp-waves which can be expressed in the light-cone string formalism. Especially, this includes the Penrose limit of giant gravitons and we will examine them in detail.

[13]Even though the first solution (figure 1) has also a discontinuity at $y = 0$, the D3-brane world-volume can be described by the 'double cone' and is not unnatural. It is also easy to see that the discontinuity does not contribute to the energy by an explicit calculation.

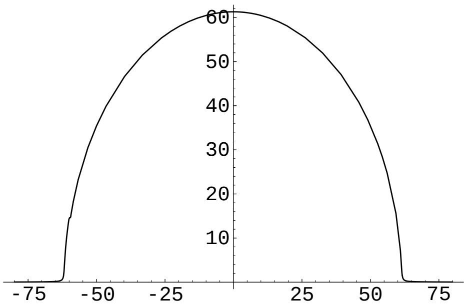

Figure 2: The behaviour of a solution $R(y)$ in the case $R_{\max}(= 62) > R_0$. Here we scaled y and $R(y)$ such that $\alpha = 1$ as before. This solution is not exactly left-right symmetric in spite of its appearance. The derivative of $R(y)$ is not continuous at $y = y_0(\simeq -59.5)$. The region $y > y_0$ corresponds to the minus sign in (3.14) and the one $y < y_0$ to the plus sign.

In the type-IIB theory on the flat background, the BPS D-branes are (D(-1),D1,D3,D5, D7,D9) and we can put these D-branes at any points preserving 16 supersymmetries out of 32. On the other hand, in the pp-wave background, which has also 32 supersymmetries, all of these D-branes are not BPS due to its curved background and RR-flux [12] (see also [11, 23, 30]). Naively, BPS D-branes are pinned at the origin because the RR-flux acts as the mass term. However, it can move around without losing the supersymmetries if we assume gauge flux on D-branes as first pointed out in the example of D5-brane [17] (see also [27] for the similar issue on BPS D-branes in the PP-wave limit of $AdS_3 \times S^3 \times T^4$). This is very natural since this background is Lorentzian symmetric space in spite of its appearance in the light-cone gauge. Below we conduct the complete investigation of supersymmetric D-branes on the pp-waves in terms of the open string boundary conditions, which includes a series of the Penrose limit of giant gravitons as D3-branes.

The string action on this pp-wave background can be described in the GS formalism in the light cone gauge [3]. Then we are restricted to open strings between D-branes whose boundary conditions in the light cone directions X^+, X^- are both Neumann, i.e., $(+, -, m, n)$ D-branes. The notation $(+, -, m, n)$ means the D-brane whose Neumann Directions are the two light-cone directions X^+, X^-, m ones in $X^1, \dots, X^4$ and n ones in $X^5, \dots, X^8$ following the convention in [17]. In the light-cone gauge the original 32 supersymmetries are divided into 16 kinematical and 16 dynamical supersymmetries, where the half BPS D-branes in the light-cone gauge preserve 8 out of each. Under this condition we can find that D-branes which preserve partial supersymmetries are *only* those in the table 1 [12, 17].[14]

Except D1-branes, all dynamical supersymmetries are broken if the D-branes are away from the origin. On the other hand, the D1-branes keep 8 dynamical ones even if these are away from the origin.

However, it may be possible that these D-branes can be moved away from the origin with preserving their 8 dynamical SUSY by adding the gauge flux on them or their boosting.

[14]It was pointed out in [23] that the D-branes which preserve less than half supersymmetries are inconsistent because they do not seem to have the open-closed modular property.

	World-Volume	Dynamical SUSY	Kinematical SUSY	Total
D1	$(+,-,0,0)$	8	0	8
D3	$(+,-,2,0),(+,-,0,2)$	8	8	16
D5	$(+,-,3,1),(+,-,1,3)$	8	8	16
D7	$(+,-,4,2),(+,-,2,4)$	8	8	16

Table 1: D-branes placed at the origin.

	World-Volume	Dynamical SUSY	Total	Condition	Movable directions
D3	$+,-,7,8$	8	16	boost in $5,6$	$5,6$
D5	$+,-,4,6,7,8$	8	16	flux in 4	5
D7	$+,-,3,4,5,6,7,8$	4	12	boost or flux	$1,2$

Table 2: D-branes placed away from the origin.

In our approach, the flux and boost can be treated easily because these appear only as the change of the boundary condition. We will find in section 4.2 that the result is described by the table 2.[15]

The symbol 'flux in I' means the addition of the proper gauge flux F_{+I} on the D-brane. A D5-brane with the gauge flux can be placed away from the origin keeping 16 supersymmetries as already pointed out in [17] in a somewhat different approach. Our new result here is that a half BPS D3-brane is also movable with the appropriate boosting. We will find that this rotating D3-brane corresponds to the giant graviton which is discussed in the section 2. On the other hand, a half BPS D7 is not movable without breaking 4 supersymmetries (movable D7 totally preserves 12 ones).

We can also see the effect of the gauge flux or boosting in the open string spectrum. When we consider the 1/2 BPS D-brane without flux and boost, we can find that the open string spectrum on this brane has the mass term $\sim \mu^2 x^2$ when the brane is moved to $x \neq 0$. On the other hand, when we consider 1/2 BPS rotating D3-brane, the open string spectrum which is calculated in the rotating coordinates [46] does not have the mass term. We can also obtain the similar result in the case of the 1/2 BPS D5-brane with the proper flux. It would be very interesting if we can compare this spectrum with the gauge theoretic results.

4.1 String action and supersymmetry

We mainly follow the notation of Metsaev and Tseytlin [4]. The string world-sheet action is

$$S = \frac{1}{2\pi\alpha'}\int \mathrm{d}\tau \int_0^\pi \mathrm{d}\sigma \left[\frac{1}{2}\partial_+ X^I \partial_- X^I - \frac{1}{2}m^2 (X^I)^2 + \right.$$
$$\left. + \sqrt{2}\left[i(\bar{S}\partial_\tau S + S\partial_\tau \bar{S} + S\partial_\sigma S + \bar{S}\partial_\sigma \bar{S}) - 2m\bar{S}\Pi S\right]\right], \quad (4.1)$$

[15] Though we treat only the D-branes of the special directions in this table for simplicity, the general results can be easily reproduced by the background $SO(4) \times SO(4)$ and $\mathbb{Z}_2 (1234 \leftrightarrow 5678)$ symmetry.

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where we use the light cone gauge $X^+ = p^+\tau$ and define $m = \mu p^+$, $\partial_\pm = \partial_\tau \pm \partial_\sigma$. S is an 8 dimensional complex spinor and $\bar{S}$ is its conjugate. We defined $\Pi \equiv \gamma^1\gamma^2\gamma^3\gamma^4$, where γ^I is the real and symmetric SO(8) γ matrix. The equations of motion are

$$\begin{aligned} \partial_+\partial_- X^I + m^2 X^I &= 0\,, \\ \partial_+ S^1 - m\Pi S^2 &= 0\,, \qquad \partial_- S^2 + m\Pi S^1 = 0\,, \end{aligned} \tag{4.2}$$

where we defined real spinors S^1 and S^2 as follows

$$S = \frac{1}{\sqrt{2}}(S^1 + iS^2)\,, \qquad \bar{S} = \frac{1}{\sqrt{2}}(S^1 - iS^2)\,. \tag{4.3}$$

Finally, the dynamical supersymmetry transformation is given by

$$\begin{aligned} \delta_\epsilon X^I &= -2i\left[\epsilon^1\gamma^I S^1 + \epsilon^2\gamma^I S^2\right]\,, \\ \delta_\epsilon S^1 &= \frac{1}{\sqrt{2}}\left[\partial_- X^I \epsilon^1\gamma^I + mX^I\epsilon^2\gamma^I\Pi\right]\,, \\ \delta_\epsilon S^2 &= \frac{1}{\sqrt{2}}\left[\partial_+ X^I \epsilon^2\gamma^I - mX^I\epsilon^1\gamma^I\Pi\right]\,, \end{aligned} \tag{4.4}$$

and the kinematical supersymmetry transformation is also

$$\delta_\kappa S^1 = \cos m\tau\kappa^1 + \sin m\tau\Pi\kappa^2\,, \qquad \delta_\kappa S^2 = \cos m\tau\kappa^1 - \sin m\tau\Pi\kappa^2\,. \tag{4.5}$$

4.2 BPS D-branes on the PP-wave background

Let us consider the open string whose both endpoints are attached to the same D-brane. The boundary conditions are the Neumann condition (N) and the Dirichlet condition (D)

$$N: \quad X'^I = 0\,, \qquad D: \quad \dot{X}^I = 0\,, \qquad \text{at} \qquad \sigma = 0, \pi\,, \tag{4.6}$$

where we defined $\dot{X}^I \equiv \partial_\tau X^I$ and $X'^I \equiv \partial_\sigma X^I$. Next, the boundary condition of fermionic fields is given by

$$S^1|_{\sigma=0,\pi} = \Omega S^2|_{\sigma=0,\pi}\,, \tag{4.7}$$

where Ω is constructed by multiplying γ^i in the Dirichlet directions as in the case of the flat background. Then we can count the number of SUSY charges which is preserved on the D-brane, by counting the number of ϵ and κ which are chosen so that δS^1 and δS^2 generated by ϵ, κ satisfy the relation

$$\delta S^1|_{\sigma=0,\pi} = \Omega\delta S^2|_{\sigma=0,\pi}\,. \tag{4.8}$$

Kinematical SUSY. First let us consider the kinematical SUSY. By using eq. (4.8) and (4.5), we find

$$\begin{aligned} &\cos m\tau(\kappa^1 - \Omega\kappa^2) + \sin m\tau(\Pi\kappa^2 + \Omega\Pi\kappa^1) = 0 \\ &\Rightarrow \kappa^1 = \Omega\kappa^2\,, \qquad (1 + \Pi\Omega\Pi\Omega)\kappa^2 = 0\,. \end{aligned} \tag{4.9}$$

Then, only the D-branes which satisfies $\Omega\Pi\Omega\Pi = -1$ always preserve 8 kinematical SUSY and the other D-branes break all kinematical SUSY. As discussed in [12], the choices of Ω which satisfies $\Omega\Pi\Omega\Pi = -1$ are only

$$\begin{aligned}
D7:&\quad \gamma^i\gamma^j\,, \qquad \gamma^{i'}\gamma^{j'}\,,\\
D5:&\quad \gamma^{i'}\gamma^i\gamma^j\gamma^k\,, \qquad \gamma^i\gamma^{i'}\gamma^{j'}\gamma^{k'}\,,\\
D3:&\quad \gamma^{i'}\gamma^{j'}\Pi\,, \qquad \gamma^i\gamma^j\Pi'\,,
\end{aligned} \tag{4.10}$$

where we defined $i,j,k = 1,\ldots,4$, $i',j',k' = 5,\ldots,8$ and $\Pi' \equiv \gamma^5\gamma^6\gamma^7\gamma^8$. Notice that D1 ($\Omega = \Pi\Pi'$) and D9 ($\Omega = 1$) break all kinematical SUSY.

Dynamical SUSY. Next let us consider dynamical SUSY. By using eq. (4.4), (4.6) and (4.8), we obtain the condition

$$(\partial_- X^I\gamma^I\epsilon^1 + mX^I\Pi\gamma^I\epsilon^2) = \Omega(\partial_+X^I\gamma^I\epsilon^2 - mX^I\Pi\gamma^I\epsilon^1)\,, \qquad \text{at} \qquad \sigma = 0,\pi \tag{4.11}$$

and this is rewritten as follows

$$(\dot{X}^a\gamma^a - X'^i\gamma^i)(\epsilon^1 - \Omega\epsilon^2) + mX^I(\Pi\gamma^I\epsilon^2 + \Omega\Pi\gamma^I\epsilon^1) = 0\,, \tag{4.12}$$

where we have used the index 'a' as Neumann directions and 'i' as Dirichlet. This relation must be satisfied at any $\dot{X}^a, X'^i$ and X^a and thus we obtain

$$\epsilon^1 = \Omega\epsilon^2\,, \qquad mX^a\Pi(1+\Pi\Omega\Pi\Omega)\gamma^a\epsilon^2 + mX^i\Pi(1-\Pi\Omega\Pi\Omega)\gamma^i\epsilon^2 = 0 \tag{4.13}$$

is the condition. Namely, only the D-branes which satisfy $\Pi\Omega\Pi\Omega = -1$ and $X^i = 0$ preserve half dynamical SUSY and otherwise all dynamical SUSY is broken.

Notice that D1-brane ($\Omega = \Pi\Pi'$) is exceptional because there is no X^a direction. D1-brane satisfies eq. (4.13) at any X^i, i.e., half dynamical SUSY is preserved at any X^i.

Dynamical SUSY with Gauge flux and boost. As we saw, D3,D5 and D7-branes which satisfy $\Pi\Omega\Pi\Omega = -1$ preserve 8 dynamical SUSY charges only when these are at the origin. It may be possible, however, that these D-branes are moved from the origin with preserving their 8 dynamical SUSY by adding the gauge flux and boosting, because we can change the boundary condition with the gauge flux and boost. With the gauge flux $F_{\mu\nu}$ and boost v_i, we can shift the boundary condition to

$$0 = X'^a - F_{a+}\partial_\tau X^+ = X'^a - p^+F_{a+}\,, \qquad 0 = \dot{X}^i - v_i\partial_\tau X^+ = \dot{X}^i - p^+v_i\,, \tag{4.14}$$

and then eq. (4.13) changes to

$$\epsilon^1 = \Omega\epsilon^2\,, \qquad \left[A_I\gamma^I\Omega + mX^i\Pi\gamma^i\right]\epsilon^2 = 0\,, \qquad A_I\gamma^I \equiv p^+(-F_{a+}\gamma^a + v_i\gamma^i)\,. \tag{4.15}$$

All we have to do is to study this equation for each of D3,D5,D7-branes. Let us study D3-brane first because we are especially interested in giant gravitons.

When D3-brane is extended in $+,-,7,8$ directions ($\Omega = \Pi\gamma^5\gamma^6$), eq. (4.15) is

$$\epsilon^1 = \Omega\epsilon^2\,, \qquad \left[A_I\gamma^I + m\gamma^5\gamma^6(X^1\gamma^1 + \cdots + X^6\gamma^6)\right]\epsilon^1 = 0\,. \tag{4.16}$$

Eq. (4.16) is satisfied for any ϵ^1 when we set

$$A_5 = -mX^6 \,, \qquad A_6 = mX^5 \,, \qquad X^1 = X^2 = X^3 = X^4 = 0 \,. \tag{4.17}$$

Namely, D3-brane can move X^5, X^6 directions freely with preserving the half SUSY by boosting in the X^5 and X^6 directions. We can interpret this D-brane as a D3-brane rotating in the $x^5 - x^6$ plane. This D-brane just coincides with the Penrose limit of the giant graviton (2.8) discussed[16] in the section 2. The ratio of the radius r to velocity v is given by

$$\frac{r}{v} = \frac{\sqrt{(X^5)^2 + (X^6)^2}}{v} = \frac{1}{\mu} \,, \tag{4.18}$$

and this is indeed the same as in (2.8).

The issue of whether the brane is movable or not will also be important when we consider orbifold theories of the pp-waves. If we consider the Penrose limit of $AdS_5 \times S^5/Z_M$ orbifold as in [47], then we can naturally identify each of M types of fractional D3-branes [48] in the pp-wave with the dibaryon operators

$$B^i = \det Q_i \,, \qquad (i = 1, 2, \ldots, M) \,, \tag{4.19}$$

where Q_i denotes the complex scalar in the direction $x^5 - x^6$ which corresponds to the i-th arrow of the quiver diagram. Note that the subdeterminant operator like (2.9) is not allowed in this case because of the requirement of the gauge invariance. This is consistent with the fact that fractional D-branes are pinned at the fixed point. If we consider the superposition of M different fractional D-branes, then we can put it away from the fixed point. The gauge dual of this fact is that we can construct the (gauge invariant) determinant operator $\det Q = \prod_{i=1}^{M} \det Q_i$ and the subdeterminant operators. It may also be interesting to consider the correspondence between open string in the pp-wave and gauge theory as in [49, 18].

We can also examine D5 and D7-branes similarly. A D5-brane ($\Omega = \gamma^5\gamma^1\gamma^2\gamma^3$) with the gauge flux $p^+F_{4+} = mX^5$ can move in the X^5 direction preserving 8 dynamical SUSY. This fact was already pointed out in terms of the world-volume theory in [17] and can be obtained the Penrose limit of $AdS_4 \times S^2$ brane found in [50]. A D7-brane with any flux and boost can not move from the origin with preserving 8 dynamical SUSY, but it is easy to see that preserving 4 dynamical SUSY is possible.

5. Conclusions and discussions

In this paper we have examined the giant gravitons in the pp-wave background from several viewpoints. In the background they can be described by spherical D3-branes whose world-volume includes only x^+ direction or D3-brane planes whose world-volume includes both x^+ and x^-. With respect to the former giant graviton we computed the correlators in the dual gauge theory and found that they are finite in this limit and show their interesting exponential behaviour.

[16] With respect to the D3-brane at the origin this interpretation was first discussed in [18]. Here we extend this result into that away from the origin.

We also see that the former excitation of giant graviton can appear in the system of multiple D1-branes and indeed they give moduli of the non-abelian D1-branes. It would be interesting to study the dual Yang-Mills picture of this phenomenon.

The latter giant graviton can be described by the world-sheet theory in the light-cone gauge. We showed that the corresponding D3-brane should be boosted in order to preserve sixteen supersymmetries. This result just matches with the Penrose limit of giant gravitons. We also computed the open string spectrum and observed that the zero-mode mass term vanishes.

There is another world-sheet description of D-branes, i.e. boundary states [11, 23]. Since all of these D-branes are space like in the light-cone gauge, they are completely different from those discussed in this paper. Because the boundary state can be expanded in terms of closed string modes, we may regard it as a coherent sum of BMN operators. The intriguing point is that even though the massive modes are included in the boundary states, they can preserve the partial supersymmetry. It would be interesting to understand the gauge theoretic interpretation of this.

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A. BPS equation for D3-brane in PP-wave background

Here we would like to derive the BPS equation for a D3-brane in the pp-wave background. First we briefly review the supersymmetry which is preserved by a Dp-brane following the paper [17] (see also the references therein). The projector Γ ($\Gamma^2 = 1$) is defined as

$$(d\xi)^{p+1}\Gamma = -\frac{1}{\sqrt{-\det(G+F)}}\; e^F \wedge (\oplus_{n\in \text{even}}\Gamma_{(n)}K^n I)\,, \tag{A.1}$$

where $\Gamma_{(n)}$ is given by

$$\Gamma_{(n)} = (\,\mathrm{d}\xi^n \wedge \mathrm{d}\xi^{n-1} \wedge \cdots \wedge \mathrm{d}\xi^1)\partial_{i_1}X^{m_1}\partial_{i_2}X^{m_2}\cdots\partial_{i_n}X^{m_n}\Gamma_{m_1 m_2\cdots m_n}\,. \tag{A.2}$$

The operation K is the complex conjugation and I is the multiplication of $-i$. A spinor ϵ in the type-IIB theory can be written as $\epsilon = \epsilon_1 + i\epsilon_2$, where ϵ_1 and ϵ_2 are the (16 components) Majorana-Weyl 10D spinor. The number of the preserved supersymmetries is given by that of the Killing spinors ϵ which are invariant under Γ.

Now we are interested in the maximally supersymmetric pp-wave (we set $\mu = 1$ below for simplicity)

$$ds^2 = -2dx^+dx^- - \left(\sum_{a=1}^{8}(x_a)^2\right)(dx^+)^2 + \sum_{a=1}^{8}(dx_a)^2. \tag{A.3}$$

We denote the gamma matrix (as in (A.2)) in this background as Γ_μ and one in the tangent space as γ_μ. Then we obtain

$$\begin{aligned} \Gamma_+ &= \gamma_+ + \frac{|x|^2}{2}\gamma_- \,, \qquad \Gamma_- = \gamma_- \,, \\ \{\gamma_+, \gamma_-\} &= -2 \,. \end{aligned} \tag{A.4}$$

The Killing spinor ϵ in this background is given by

$$\begin{aligned} \epsilon = &\left(1 + \frac{i}{2}\gamma_- \left(\sum_{a=1}^{4} x^a \gamma_a \gamma_{1234} + \sum_{a=5}^{8} x^a \gamma_a \gamma_{5678}\right)\right) \times \\ &\times \left(\cos\left(\frac{x^+}{2}\right) - i \sin\left(\frac{x^+}{2}\right)\gamma_{1234}\right)\left(\cos\left(\frac{x^+}{2}\right) - i \sin\left(\frac{x^+}{2}\right)\gamma_{5678}\right)(\lambda + i\eta) \,. \end{aligned} \tag{A.5}$$

For convenience we also define

$$\psi(x^+) = \left(\cos\left(\frac{x^+}{2}\right) - i \sin\left(\frac{x^+}{2}\right)\gamma_{1234}\right)\left(\cos\left(\frac{x^+}{2}\right) - i \sin\left(\frac{x^+}{2}\right)\gamma_{5678}\right)(\lambda + i\eta) \,. \tag{A.6}$$

Let us turn to the main point. Consider the D3-brane whose world-volume is $R \times R \times S^2$, where one of Rs is the light-cone time x^+ and another is the spatial direction $y (= x^4)$ and S^2 is the sphere[17] $(x^1)^2 + (x^2)^2 + (x^3)^2 = \rho^2$. We assume the radius ρ depends only on y. In order to relate this system to M non-abelian (expanded) D1-branes we assume the $M (\in Z)$ units of magnetic flux on S^2 as $F = (\pi\alpha' M)\sin\theta d\theta \wedge d\varphi$, where we introduce the usual polar coordinate of the sphere. In this set up the projection is written as ($'$ means ∂_y and $\alpha \equiv \pi\alpha' M$)

$$\begin{aligned} \Gamma = &\frac{-1}{\sqrt{(1+\rho'^2)(y^2+\rho^2)(\rho^4+\alpha^2/4)}}\left(-\gamma_+ + \frac{\rho^2+y^2}{2}\gamma_-\right) \times \\ &\times \left(\gamma_{234}\rho^2 + \gamma_{123}\rho'\rho + \gamma_4 K\frac{\alpha}{2} + \gamma_1 \rho' K \frac{\alpha}{2}\right) I \,. \end{aligned} \tag{A.7}$$

Next it is easy to see that we must assume

$$\gamma_+ \psi(x^+) = 0 \,, \qquad \text{(or equally} \qquad \gamma_+\lambda = \gamma_+\eta = 0) \,, \tag{A.8}$$

which means that the preserved supersymmetries are all the dynamical ones. Then we obtain

$$\begin{aligned} \Gamma\epsilon = &\frac{i}{\sqrt{(1+\rho'^2)(y^2+\rho^2)(\rho^4+\alpha^2/4)}}\left(\frac{\rho^2+y^2}{2}\gamma_- + i\left(\gamma_{234}\rho - \gamma_{123}y\right)\right) \times \\ &\times \left(\gamma_{234}\rho^2 + \gamma_{123}\rho'\rho^2 - \gamma_4 K\frac{\alpha}{2} - \gamma_1 \rho' K \frac{\alpha}{2}\right) \psi \,. \end{aligned} \tag{A.9}$$

Furthermore by using (A.8) we can find $(\gamma_{1234} + \gamma_{5678})\eta = (\gamma_{1234} + \gamma_{5678})\lambda = 0$. Thus we can identify $\psi(x^+)$ with $\lambda + i\eta$. Then the projection equation is rewritten as

$$\begin{aligned} &\frac{1}{\sqrt{(1+\rho'^2)(y^2+\rho^2)(\rho^4+\alpha^2/4)}}(\gamma_{234}\rho - \gamma_{123}y) \times \\ &\times \left(\gamma_{234}\rho^2 + \gamma_{123}\rho'\rho^2 - \gamma_4 K\frac{\alpha}{2} - \gamma_1 \rho' K \frac{\alpha}{2}\right)(\lambda + i\eta) = -(\lambda + i\eta) \,. \end{aligned} \tag{A.10}$$

[17] The relation to R discussed in the context of D1-branes is given by $\alpha R/2 = \rho$.

or for each component

$$\left[(y\rho' - \rho)\rho^2 + \frac{\alpha}{2}(y + \rho\rho')\gamma_{1234} + (y + \rho\rho')\rho^2\gamma_{14} + \frac{\alpha}{2}(y\rho' - \rho)\gamma_{23}\right]\lambda =$$
$$= -\sqrt{(1 + \rho'^2)(y^2 + \rho^2)\left(\rho^4 + \frac{\alpha^2}{4}\right)}\lambda$$
$$\left[(y\rho' - \rho)\rho^2 - \frac{\alpha}{2}(y + \rho\rho')\gamma_{1234} + (y + \rho\rho')\rho^2\gamma_{14} - \frac{\alpha}{2}(y\rho' - \rho)\gamma_{23}\right]\eta =$$
$$= -\sqrt{(1 + \rho'^2)(y^2 + \rho^2)\left(\rho^4 + \frac{\alpha^2}{4}\right)}\eta\,. \tag{A.11}$$

Finally we can conclude that the background preserves eight (dynamical) supersymmetries such that

$$\gamma_{1234}\lambda = \pm\lambda\,, \qquad \gamma_{1234}\eta = \mp\eta\,, \tag{A.12}$$

if the following BPS equation is satisfied

$$2\rho^2(y + \rho\rho') = \pm\alpha(y\rho' - \rho)\,, \tag{A.13}$$

where we should choose[18] $+$ $(-)$ sign if $y+\rho\rho' < 0$ (> 0). If we consider a D3-brane with no-flux $M = 0$ and require the BPS equation $y + \rho\rho' = 0$, then the background preserves all of the dynamical supersymmetries (1/2BPS). This solution is given by the three dimensional sphere $y^2 + \rho^2 =$ const. and is no other than the giant graviton as discussed in [5, 17, 26].

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